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Numerical treatment of three-dimensional particulate flows using mixture models

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We address the numerical simulation of mixture models for (semi-) dilute and concentrated suspensions of particles in incompressible fluids in three-dimensions [1]. Averaged continuum models in which the effective density and viscosity of the mixture depend on the local volume fraction of the disperse phase are considered [2, 3]. The velocity and pressure of the suspension are given by the incompressible Navier-Stokes equations. The model is closed by problem-dependent constitutive laws. In the dilute regime, we use an analog of the Boussinesq approximation for natural convection flows [4, 5].

The numerical implementation of the presented mixture model is based on the methodology for buoyancy-driven turbulent bubbly flows [6]. The generalized Navier-Stokes system and the continuity equation for the volume fraction of the disperse phase are discretized using an implicit high-resolution finite element scheme [7, 8] implemented in the parallel 3D version of the open-source finite element library FeatFlow [9]. The velocity and pressure are approximated using the stable Q_2P_1 pair on hexahedral meshes. The continuity equation for the volume fraction of the disperse phase is discretized using Q_1 elements. The coupling with the Navier-Stokes solver is accomplished using outer iterations. Steady-state solutions are calculated using pseudo-time-stepping. The maximum principle for volume fractions is enforced using algebraic flux correction [10-12]. To prevent the volume fractions from exceeding the maximum packing limit, a conservative overshoot limiter is applied to the converged convective fluxes at the end of each time step.

The flux-corrected transport (FCT) algorithm proposed in [10] combines the idea of Leiderman, Fogelson [11] with algebraic flux correction [11,12]. The discretized convective term is decomposed into numerical fluxes and the magnitude of these fluxes is limited so as to get rid of unrealistic maxima. The advantages of constraining the discrete solution in this way are twofold. First, there is no need for tuning any free parameters or choosing the 'right' damping function for the convective flux. Second, the employed limiting strategy does not prevent the particles from leaving the regions of maximum concentration.

A numerical study of the proposed approach is performed for flows over a backward-facing step and in a lid-driven cavity. The numerical results for two test problems illustrate the ability of the proposed scheme to handle dilute and concentrated suspensions

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