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Weight estimation of Supersonic Transport Aircrafts

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From the dawn of aviation era until today, human has always imagined to fly faster and higher. Moreover, in the fast moving world of 21st century, supersonic aircrafts have developed deeper interests in mankind. After the first generation supersonic passenger aircrafts were grounded due to their high noise level, large fuel consumption and poor aerodynamic efficiency, leading aerospace organizations and researchers all over the world are focusing on investigation of exotic configurations for second generation supersonic civil aircrafts that must overcome all of these problems. The Oblique Flying Wing (OFW) and Lifting Body (LB) configurations show promising attitude to be next generation of economic supersonic passenger aircraft and thus are chosen for investigation. This article presents the weight estimation procedure and the main parameters leading to the design and affecting the take-off weight for these configurations, with a payload of 300 passengers. A Logarithmic regression model is determined to link between empty and take-off weights. Also a comparison between the two configurations is done to understand the sensitivity of take-off weight on different parameters. It is found that the empty weight of the LB is less than the OFW by 16.6% while the take-off weight of the OFW is less by 18.6%. Fuel percentage for the OFW is 42.46% of its take-off weight compared to 57.87% for the LB.

1. Introduction

Supersonic transport aircrafts are of great interest to aerospace researchers and also for civilians since mid-twentieth century. The curiosity and interest of humans towards supersonic civil planes have been increased tremendously since the time they have tasted the indescribable experience of flying supersonically in Concorde and TU-144. Since the time these planes have been removed from service due to their high level of sonic boom signature and large fuel consumption, many aerospace organizations and renowned researchers across the world are focusing on different types of special configurations (as Joint wing, Lifting body, Oblique flying wing) to investigate and reduce the level of sonic boom and improve the aerodynamic efficiency. To make improvements in the fuel consumption efficiency, various engines are also being investigated.

This paper is first in series in the attempt to design an economical supersonic civil transport aircraft with low sonic boom and high aerodynamic characteristics. For this process, the oblique flying wing and lifting body configurations are selected. This article presents the weight estimation process and the effect of driving parameters on take-off and empty weights for these configurations.

1.1 Oblique Flying Wing Configuration

The basic concept of the OFW as introduced by R.T. Jones in 1958 [1] is a straight wing with variable sweep angle. This sweep angle varies with the flight speed to maintain subsonic airflow normal to the wing leading edge. This ensure a good performance of the aircraft in subsonic and supersonic regimes. The main three features of this configuration are (1) unsymmetrical sweep, (2) uniform lift distribution over the wing, and (3) the variable sweep angle [2]. Unsymmetrical sweep makes lift to be distributed over twice the wing length compared to symmetric sweep as shown in Figure 1. This leads to reduction of volume dependent wave drag and lift dependent wave drag. The uniform lift distribution has an important role in sonic boom reduction since the equivalent body of revolution is more uniform. Due to the variable sweep angle, the aircraft has outstanding aerodynamic characteristics as achieving very high Lift-to-Drag (L/D) ratio and good subsonic performance.

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Aeronautical Sciences (ICAS), Advances in Aeronautical Sciences, Vol 1., Pergamon Press, NY, 1959.

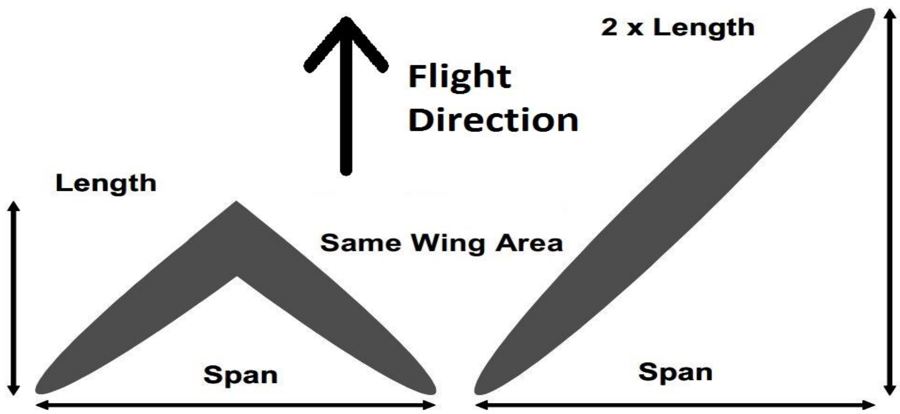

Figure 1. Comparison between swept wing and oblique wing.

This configuration has the minimum drag in both subsonic and supersonic regimes. In subsonic regime, the induced drag (drag due to lift) is inversely proportional to the span. The aircraft in this regime will fly without sweep, which allows it to use its whole length as a span. In supersonic regime wave drag arise, which is dependent on the lift distribution over the aircraft length. Since this configuration has twice the length compared to the conventional swept wing, the lift dependent wave drag is reduced by factor of 4 and the volume dependent wave drag by factor of 16.

Concerning the sonic boom, the OFW can easily achieve pressure increment of 50-80 N/m^2 depending on its geometry, weight and altitude. This means at least one third less than the 1st generation. The lateral distribution of the sonic boom signature is highly dependent on the azimuth. The interference between the equivalent areas due to lift and volume causes cancellation of the aft-shock [3].

1.2 Lifting body Configuration

The first generation of supersonic commercial aircrafts failed in many areas, and that mainly includes the high level of sonic boom level, low L/D ratio and not economical (high travel cost per passenger). This supersonic lifting body aircraft designs aims to overcome these disadvantages.

Conventional configurations have very high value of sonic boom and don’t have possibility of flying supersonically over land, but the unconventional lifting body configuration might produce reduced N-wave signature or sonic boom signature different from N-wave, that might not be harmful to human hearing and will be accepted over land. Lifting body aircraft configuration has lift distribution over the whole length of the aircraft. Thus, by changing the lift distribution over the nose and the wing part, the corresponding pressure distribution and hence the corresponding sonic boom produced by the nose and the wing lift can be changed and the trivial N-wave sonic boom signature can be altered.

The lifting body configuration has very good aerodynamic characteristics with increased lift due to lift produced by fuselage part and thus achieving high value of L/D ratio. In this design, wing plays some role of fuselage and fuselage plays some role of wing. Thus due to high lifting capacity and large space due to blended wing-body design, it aims for a payload of 300 passengers, three times the payload of Concorde while having only twice the take-off weight. Thus, it will help in reducing the per person travel cost up to 33% and making it more affordable for civilians. A preliminary design of the configuration is presented in Figure 2.

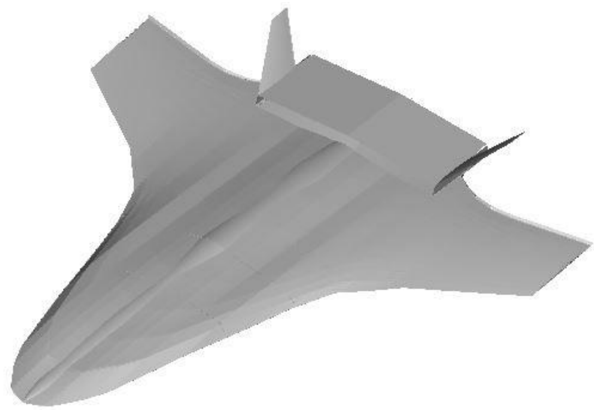

Figure 2. Preliminary design of lifting body configuration.

2. Weight estimation procedure

Designing an aircraft is an iterative procedure. It is important to predict the weight of the aircraft that will carry the payload and meet the mission requirements. So, obviously the first step in the process is to estimate the mass of the aircraft and its decomposition.

The procedure for weight estimation is started by building the regression model for empty and take-off weights of similar aircrafts and get logarithmic relation between them. Then the payload weight according to the requirements followed by initial guess for the take-off weight is calculated. The fuel weight is then calculated according to the mission profile. Then the empty weight is calculated from the mass conservation which provide linear relation between empty and take-off weights. These two equations must be solved by iteration till convergence takes place. This procedure is summarized in Figure 3.

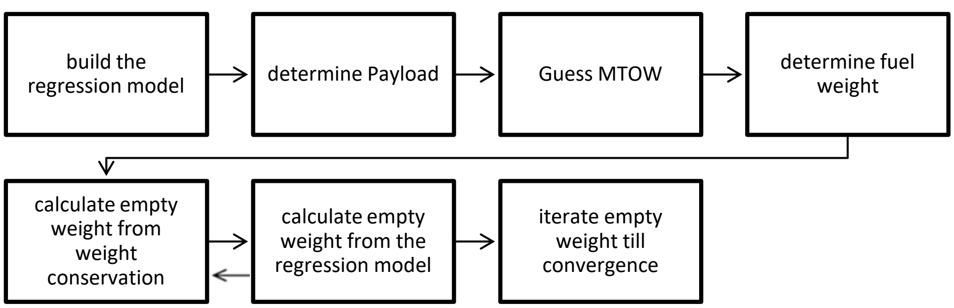

Figure 3. Weight estimation procedure.

2.1 Regression model

A regression model is constructed to link the empty and take-off weights by using the linear relationship that exists between $\log_{10} W_e$ and $\log_{10} W_{TO}$. The regression model is built by using a set of similar aircrafts configurations and mission. After processing, the model is casted to the following equation

$$\log_{10} W_e = A + B \log_{10} W_{TO} \tag{1}$$

and the coefficients A and B must be optimized to give the minimum error (maximum coefficient of determination).

For OFW, the works of Waters M. et al [4], Van der Velden A. [5] and Hirschberg et al [6] are used to construct the regression model. The data is summarized in Table 1.

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Commented [me3]: Vandervelden, A. J. (1989). The conceptual design of a Mach 2 Oblique Flying Wing supersonic transport.

Commented [me4]: Hirschberg, Michael J., David M. Hart, and Thomas J. Beutner. "A summary of a half-century of oblique wing research." *AIAA Paper* 150 (2007): 2007.

Table 1. Aircraft data used to construct regression model for OFW

Take-off Weight	Empty Weight	Fuel	Passengers
392357	181039	156155	484
304200	130200	139400	---
858881	395883	401888	291
1074146	488353	493393	440
1220645	553214	553191	544
442252	217724	---	708

It is found that for the OFW, A=-0.338 and B=1.007 and the coefficient of determination is 0.9998 which shows very good alignment with the data as shown in Figure 4.

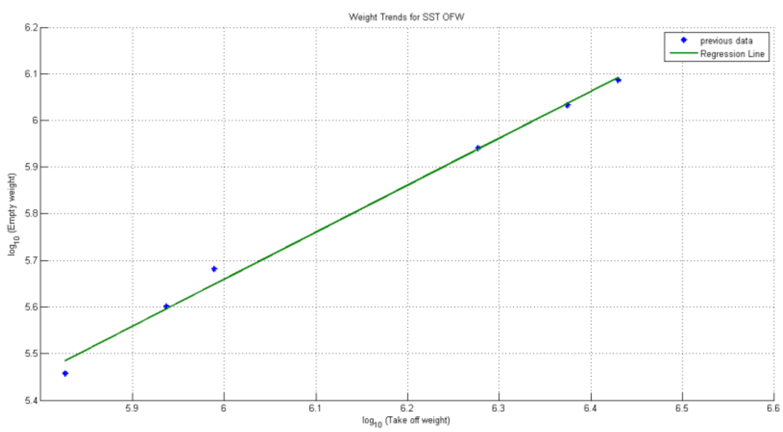

Figure 4. Logarithmic regression model for the OFW

For the LB, the coefficients A and B are considered based on previous database provided by Roskam [7] for the conventional delta wing configurations and then examined by two LB designs developed by NASA [8][9], the data being summarized in Table 2. The coefficients A and B used are -0.338 and 1.007 respectively with coefficient of determination of 0.9982 which shows very good alignment with the data. After adding data from NASA designs, the coefficient of determination reduced by 0.05% which means that for the preliminary calculations, the LB configuration coincides with the delta wing configuration as shown in Figure 5.

Table 2. Aircraft data used to construct regression model for LB

Aircraft	Take-off Weight	Empty Weight	Fuel	Passengers
Concorde	389000	172000	202809	100
TU 144	396830	187400	209440	80
Boeing 969-512BA	340194	162510	155501	---
Boeing 969-512BB	750000	358270	342824	183
SM-SST	56200	25200	29880	---

Mach 3 High Sped Civil Transport Concept	713696	327796	385900	300
Mach 4 High Sped Civil Transport Concept	988088	363229	635980	250

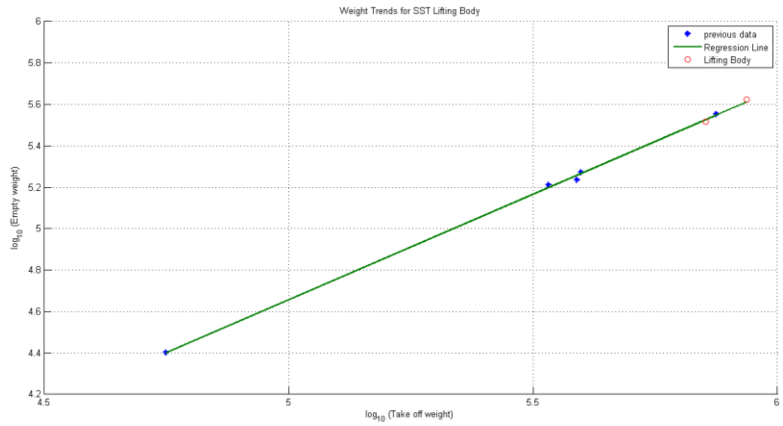

Figure 5. Logarithmic regression model for the LB

2.2 Payload

Payload is usually specified according to the requirement of the mission. For commercial flights the average mass of a person is 175 lbs with 30 lbs of luggage for short flights and 40 lbs for long flights. For the crew members, the average mass of a person is 175 lbs with 30 lbs of luggage.

2.3 Take-off weight

The take-off weight consists of the mass of the passengers, crew, fuel, and empty weight. The last two can be considered as the empty weight of the aircraft as in Equation (2)

$$W_{TO} = W_p + W_{crew} + W_F + W_{OE} \tag{2}$$

where W_{TO}, W_p, W_{crew}, W_F and W_{OE} are the take-off, payload, crew, fuel and operating empty weights respectively.

For the first iteration it is set arbitrary and then it is varied within the optimization process till it be constant and with agreement with the logarithmic regression model.

2.4 Fuel weight

The fuel weight is determined by Fuel-Fraction method. This method is based on dividing the mission into several phases as shown in Figure 6. In each phase the aircraft's mass differs from the start to the end due to fuel consumption. Based on this differences the fuel-fraction can be calculated as a ratio of the total mass of the aircraft as in Equation (3)

$$M_{ff} = W_{TO} * \prod_{1}^{N} \frac{W_n}{W_{n-1}} \tag{3}$$

Then the fuel weight is calculated by Equation (4)

$$W_F = \left(1 - M_{ff}\right) * W_{TO} \tag{4}$$

These ratios are determined by either calculations or experience. A good resource for these values could be found in [6].

2.5 Empty weight

Empty weight consists of the flight equipment, structure, and anything other than the fuel and people. It can be computed from the mass decomposition equation (2) or from the logarithmic regression model. Since both methods must obtain the same result, iteration cycle must be used to fulfil this condition.

3. Sensitivity

Since the method used to calculate the weight is highly dependent on some parameters as the mass, fuel consumption, range, lift-to-drag ratio, it is important to estimate how these parameters affect the total mass. The importance of this study is (1) To find out which parameters drive the design, (2) To provide a quick estimate of the impact of each parameter on the design, and (3) To determine which areas of technological change must be pursued, if some new mission capability must be achieved.

Analytical formulae are derived to compute the dependency of the take-off weight on the various parameters as follows:

1. Sensitivity of take-off weight to payload weight can be calculated from

$$\frac{\partial W_{TO}}{\partial W_P} = \frac{B * W_{TO}}{D - C(1-B) * W_{TO}} \tag{4}$$

where

$$C = 1 - M_{tfo} - (1 + M_{res}) * (1 - M_{ff}) \tag{5}$$

$$D = W_P + W_{crew} \tag{6}$$

M_{tfo} , M_{res}, and M_{ff} are trapped fuel, residual fuel and mission fuel fraction weights (defined in equation 3) respectively.

2. Sensitivity of take-off weight to empty weight can be calculated from

$$\frac{\partial W_{TO}}{\partial W_E} = \frac{B * W_{TO}}{10^{\frac{\log W_{TO} - A}{B}}} \tag{8}$$

3. Sensitivity of take-off weight to range can be calculated from

$$\frac{\partial W_{TO}}{\partial R} = \frac{F * C_j}{V * \frac{L}{D}} \tag{9}$$

where

$$F = -B * \frac{W_{TO} * (1 + M_{res}) * M_{ff}}{C * W_{TO} * (1 - B) - D} \tag{10}$$

4. Sensitivity of take-off weight to specific fuel consumption can be calculated from

$$\frac{\partial W_{TO}}{\partial C_j} = \frac{F * R}{V * \frac{L}{D}} \tag{11}$$

5. Sensitivity of take-off weight to lift-to-drag ratio can be calculated from

$$\frac{\partial \mathrm{W_{TO}}}{\partial \frac{\mathrm{L}}{\mathrm{D}}} = \frac{-\mathrm{F} * \mathrm{R} * \mathrm{C_j}}{\mathrm{V} * \left(\frac{\mathrm{L}}{\mathrm{D}}\right)^2} \tag{12}$$

4. Case study

To study the difference in weight between the OFW and LB more closely, a case study of a supersonic passenger aircraft is conducted. The aircraft is capable of carrying 300 passengers across the Pacific Ocean with Mach number of 1.15 over land to prevent the sonic boom on ground and Mach 2.0 over the ocean.

4.1 Mission specification and profile

The route between US and Japan across the Pacific Ocean is selected. Engine type is suggested to be turbojet engine with bypass ratio 2. Table 3 summarize the mission specifications and the mission profile is illustrated in Figure 6.

Table 3. Mission Specification

Payload	300 Passengers + luggage
Crew	2 pilots +23 air hostesses
Range	5,500 nm (31% overland and 69% overseas)
Mach number	overland: 1.15, overseas: 2.0
Altitude	overland: 36,000 ft, overseas: 52,000 ft
Engine type	Turbojet

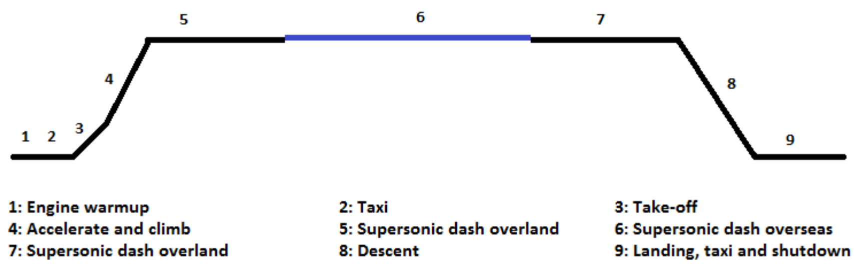

Figure 6. Logarithmic regression model for the LB

4.2 Fuel fractions

For this mission profile, the fuel fractions were obtained in the following manner and summarized in Table 4:

- Phase 1: Starting and warm up. this phase starts with W_{TO} and ending with W_1 ; W_1/W_{TO} = 0.99
- Phase 2: Taxi phase starting with W_1 and ending with W_2; W_2/W_1 = 0.995
- Phase 3: Take-off phase starting with W_2 and ending with W_2; W_3/W_2= 0.995
- Phase 4: Acceleration till Mach number 1.15 (overland). phase starting with W_3 and ending with W_4; W_4/W_3 = 0.96
- Phase 5: Cruise phase overland starting with W_4 and ending with W_5

$$R_{cr} = \frac{V}{C_j} * \frac{L}{D} * \ln \frac{W_4}{W_5} \tag{7}$$

where R_{cr} is a cruise range, C_j is the specific fuel consumption.

The range in this phase is 1593 nm, C_j of 0.7 for the OFW seems reasonable [10], L/D is set to be equal to 17 as calculated previously in [10]. Consequently, the W_5/W_4 ratio comes out to be 0.9053. For LB, C_j is estimated to be 0.9 and lift-to-drag is 10.5 based on the NASA design [8]. Thus, the W_5/W_4 ratio is 0.813.

- Phase 6: Cruise phase overseas starting with W_5 and ending with W_6

The range in this phase is 3790 nm, C_j is guessed to be 0.9, L/D is set to be equals to 10 as calculated previously in [10]. Hence, W_6/W_5 ratio comes out to be 0.7186. For the LB, C_j is estimated to be 1.2 and L/D is 9 according to the NASA design [8]. Thus, the W_6/W_5 ratio comes out to be 0.6436.

- Phase 7: Cruise overland Mach number 1.15 starting with W_6 and ending with W_7.

The range in this phase is 109 nm, and the rest of the parameters are as in Phase 5. Therefore, the ratio W_7/W_6 is 0.9932 for the OFW and 0.988 for the LB.

- Phase 8: Descend phase starting with W_7 and ending with W_8; W_8/W_7 = 0.985
- Phase 9: Landing and taxi phase starting with W_8 and ending with W_9, W_9/W_8 = 0.992

Table 4. Fuel fractions for the specified mission

Fuel fraction	OFW	LB
W_1/W_{TO}	0.99	0.99
W_2/W_1	0.995	0.995
W_3/W_2	0.995	0.995
W_4/W_3	0.96	0.96
W_5/W_4	0.905	0.813
W_6/W_5	0.7186	0.6436
W_7/W_6	0.993	0.988
W_8/W_7	0.985	0.985
W_9/W_8	0.992	0.992

5. Results

The OFW and LB configurations are investigated for the same payload and mission. The results show that the empty weight of the OFW is 264.5 thousand lbs and 221 thousand lbs for the LB. The fuel percentage for the OFW is 42.46% of its take-off weight compared to 57.87% for the LB. The take-off weight of the OFW is 581 thousand lbs and 689 thousand lbs for the LB. The results for both the configurations are summarized in Table 5.

Table 5. Results of the weight estimation for the OFW and LB

Weight	OFW	LB
Empty	264685.16	220729.43
Fuel	246734.285	398862.4186
Payload	69625	69625
MTOW	581044.445	689216.8487

The sensitivity analysis is conducted to study and quantify the impact of variation of the design parameters on the final take-off weight. It is found that by increment of payload by 1 lb, the take-off weight increases by 8.2 lbs for the OFW and 10.5 lbs for the LB. When empty weight increases by 1 lb, the take-off weight increases by 2.2 lbs for the OFW and 3.1 lbs for the LB. In the low supersonic cruise, the take-off weight will increase for each additional nautical mile range by the value of 114.2 lbs for the OFW and 488 for the LB while for the high supersonic cruise, the take-off weight increment is 143.5 lbs for the OFW and 436.5 for the LB. The specific fuel consumption is a critical value and can affect the take-off weight very much. When L/D ratio is increased by 1, then take-off weight of the OFW for the low supersonic cruise is reduced by 11.4 thousands lbs and by 54.4 thousands lbs for the high supersonic cruise. On the other hand, for the LB the reduction in take-off weight is 79.1 thousands lbs for the low supersonic cruise and 183.8 thousands lbs for the high supersonic cruise. The results of the sensitivity analysis (change in MTOW with respect to change in 1 unit of given parameters) are summarized in Table 6.

Table 6. Sensitivity Analysis

Parameters (changing by 1 unit)	Change in MTOW for OFW	Change in MTOW for LB	Unit
Payload	8.235	10.459	lb/lb
Empty weight	2.21	3.0837	lb/lb
Overland range	114.177	487.993	lb/nautical mile
Overseas range	143.496	436.483	lb/nautical mile
Overland fuel consumption	277747.5	923296.5	lb/(lb.lb.hr)
Overseas fuel consumption	604304.2	1378620	lb/(lb.lb.hr)
Overland lift-to-drag ratio	-11436.7	-79139.7	lb
Overseas lift-to-drag ratio	-54387.4	-183816	lb

6. Conclusion

Despite the fact that the empty weight of the LB is less than the OFW by 16.6%, the take-off weight of the OFW is less by 18.6% due to the less fuel consumption and the better aerodynamic efficiency of the OFW. This advantages are evident in the fuel consumption during the flight as fuel percentage for the OFW is 42.46% of its take-off weight compared to 57.87% for the LB.

The take-off weight of the LB is more sensitive than OFW to any variation in the input parameters. This means that the LB weight will vary significantly after calculating the accurate values through the design loop. On the other hand, take-off weight of the OFW will not show severe variation in the design loop. Both configurations show nearly the same sensitivity for the variation of payload and empty weights. Considering the range sensitivity, it is better for the OFW to fly overland while for the LB flying overseas is more beneficial. The LB shows better response for the enhancement of the L/D ratio compared to the OFW. And hence, the enhancement in L/D ratio for the LB must be taken into account to reduce the weight. Reduction of the fuel consumption is obviously has a favorable effect on the aircraft weight especially for the LB.

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